

Improving Radial Triangulation-based Forensic Palmprint Recognition According to Point Pattern Comparison by Relaxation

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Abstract

Forensic palmprint recognition, which mainly deals with high-resolution palmprints and latent-to-full palmprint comparison, has aroused research highlights because of the increased use of the evidence of palmprints in forensics. There are some in-depth works on high-resolution palmprint preprocessing (i.e., segmentation and enhancement) and feature extraction. However, few works on latent-to-full palmprint comparison have been done. Recently, radial triangulation-based latent-to-full palmprint comparison algorithm was proposed as it has been proposed for forensic likelihood ratio computation using fingerprints, and proved to have identification usability and efficiency for palmprint comparison. In this work, we generalize point pattern comparison by relaxation to minutiae-based palmprint recognition and improve the latent-to-full palmprint comparison algorithm based on radial triangulation. Firstly, local minutiae comparison is modified according to novel point pattern comparison method and global minutiae comparison is based on centroid distribution. Then logistic regression learning is used for comparison score computation. Performance of the proposed algorithm is evaluated on forensic databases including 22 latent palmprints from real cases and 8680 full palmprints from criminal investigation field. Experimental results show the improvement on identification accuracy and efficiency of our approach. A rank-1 identification rate of 69% is achieved, compared with 63% of previous radial triangulation-based.

1. Introduction

The interest in forensic palmprint recognition has been aroused in the last year due to the increase in the forensic cases where palmprint recognition has yielded important information about identity. It mainly deals with high-resolution palmprints, where minutia-based palmprint recognition is mainly the type of latent-to-full palmprint comparison. There are some in-depth works on high-resolution palmprint preprocessing (i.e., segmentation [15] and enhancement [16]) and feature extraction [1]. However,

few works on latent-to-full palmprint comparison have been done. Pioneering works such as MinutiaCode-based approach [2] made relevant contribution to the objective of real forensic latent-to-full palmprint comparison. However, the approach has two main drawbacks: 1) MinutiaCode is not robust to distortion and disturbance of central minutiae. 2) MinutiaCode-based comparison is still time-consuming, while comparison efficiency is significant for forensic automatic palmprint identification systems, for instance in large-scale database queries. Therefore, more research is needed in this field in order to achieve a robust and meaningful technology.

Moreover, the interpretation needs of modern forensic science [12] calls for a rigorous quantification of the evidential value of biometrics, such as fingerprints and palmprints. Recently, radial triangulation has been proposed as a step towards this objective in fingerprints [3], and used for modeling minutiae manually extracted by forensic experts. Based on this approach, distances among minutiae models are used as a basis for the computation of likelihood ratios, which represent the value of the evidence in a given case. Motivated by [3, 4], radial triangulation-based forensic palmprint recognition algorithm [10] was proposed and proved to have identification usability and efficiency. However, some problems remain in [10]: 1) The discriminative power of the similarity based on radial triangulation structure is not stable. 2) To select more accurate calibration centers, finer alignment based on global information is needed. 3) Learning based comparison score computation is needed to better distinguish between genuine and imposter.

In fact, the minutiae comparison problem can be viewed as a point pattern comparison problem [13] which plays a central role in many pattern recognition and computer vision tasks, and extensively studied yielding families of approaches known as algebraic geometry, Hough transform, relaxation, Operations Research solutions, energy-minimization. Previous work [5] indicated that point pattern comparison by relaxation was tolerant to global distortion and had some successful applications [6-9].

In this work, we generalize point pattern comparison by relaxation [5] to minutiae-based palmprint recognition and improve latent-to-full palmprint comparison algorithm based on radial triangulation. Firstly, the similarity

computation based on local minutiae comparison is modified according to novel point pattern comparison method. Secondly, alignment between latent and full palmprints based on the distribution of centroids in radial triangulation structures is implemented for global minutiae comparison. Finally, logistic regression learning is used for comparison score computation. Performance of the proposed algorithm is evaluated on forensic databases including 22 latent palmprints from real cases and 8680 full palmprints from criminal investigation field which are subsets of the databases used in [17]. Experimental results show the improvement on identification accuracy and efficiency of our approach. And rank-1 identification rate of 69% is achieved, compared with 63% of previous radial triangulation-based method and 67% of MinutiaCode-based method on the same databases.

The rest of this paper is organized as follows: Section 2 describes related works including point pattern comparison by relaxation and radial triangulation modeling. Section 3 proposes the improved latent-to-full palmprint comparison algorithm based on radial triangulation. Experimental results are provided in Section 4, with conclusions presented in Section 5.

2. Related works

2.1. Point pattern comparison by relaxation

As the minutiae comparison problem can be viewed as a point pattern comparison problem, we refer to some previous works on point pattern comparison by relaxation, especially to obtain robust solution to distortion.

It was early indicated in [5] that the relaxation approach of point pattern comparison was proved to be more tolerant to global distortion. Later some successful applications of point pattern comparison by relaxation were presented [6-9], such as comparison of constellations [6], Delaunay triangulations [8] and hand image sequence [9].

Here we generalize the relaxation approach of point pattern comparison to minutiae-based palmprint comparison. Given two minutia sets $LM = \{LM_i\}_{i=1}^m$ and $FM = \{FM_j\}_{j=1}^n$ from a latent palmprint and a full palmprint respectively, we count, for each displacement δ of LM relative to FM , how many pairs (LM_i, FM_j) lie closer together than some threshold t . When we pair LM_i with FM_j , we would like to find minutiae of the full palmprint in the same positions relative to FM_j that minutiae of the latent palmprint have relative to LM_i . Let LM_h and FM_k be any minutiae other than LM_i and FM_j , and let $\delta_{i,j}(h,k)$ be the position difference of LM_h and FM_k when LM_i is mapped into FM_j . If $|\delta_{i,j}(h,k)|$ is zero,

FM_k is exactly in the same position relative to FM_j that LM_h is relative to LM_i , so that the pair (LM_h, FM_k) should give the pair (LM_i, FM_j) maximal support; while as $|\delta_{i,j}(h,k)|$ increases, this support should decline. Let the support given to (LM_i, FM_j) by (LM_h, FM_k) be denoted by $\varphi(|\delta_{i,j}(h,k)|)$. In the experiments described in this work, we used $\varphi(x) = 1/(1+x^2)$ as [5]. For each LM_h there may be several FM_k s that lie close to it; but we only want one FM_k to correspond to LM_h when LM_i is paired with FM_j . Thus it is reasonable to define the support to (LM_i, FM_j) associated with LM_h as

$$\max_{k \neq j} [\varphi(|\delta_{i,j}(h,k)|)]. \quad (1)$$

In this way, we compute the support provided to the pair (LM_i, FM_j) by each LM_h . To obtain the total support for the pair (LM_i, FM_j) , we average contributions of all the LM_h s:

$$s_M(LM_i, FM_j) = 1/(m-1) \sum_{h \neq i} \{\max_{k \neq j} [\varphi(|\delta_{i,j}(h,k)|)]\}. \quad (2)$$

where $s_M(LM_i, FM_j)$ denotes the similarity of minutiae pair (LM_i, FM_j) .

2.2. Radial triangulation modeling

In order to execute minutiae-based comparison between latent and full palmprints, the spatial arrangement of minutiae, i.e., local minutiae structures, is essential not only for local minutiae comparison but also for global minutiae comparison, as latent prints are usually partial images.

Recently, radial triangulation was generalized to palmprint minutiae modeling [10], motivated by its application to minutiae modeling for forensic evidence evaluation in fingerprint comparison [3]. It is served as local minutiae structure as shown in Figure 1. Given a set of N minutiae, i.e., $M = \{m_k\}_{k=1}^N$ where $m_k = (x_k, y_k, \theta_k)$, the general vector form of the local minutiae structure based on radial triangulation is,

$$LS = [P_C, \{V_k, R_k, L_{k,k+1}, ST_k\}_{k=1}^N] \quad (3)$$

where $P_C = (x_C, y_C, \theta_C)$ is the centroid of the polygon, V_k is

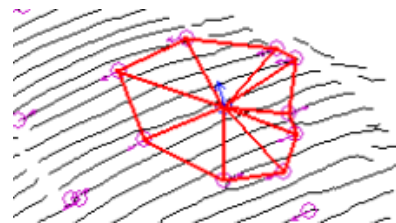


Figure 1: Radial triangulation of a set of nine minutiae from a palmprint.

a vertex where a minutia locates, R_k is the radius of minutia k to the centroid, $L_{k,k+1}$ is the length of the polygon side between minutia k and minutia $k+1$, ST_k is the area of the triangle defined by minutiae k , $k+1$, and the centroid. In order to be used in local structure comparison, each vertex V_k in a polygon has been transformed into the polar coordinate system of the centroid from its corresponding minutiae m_k as following:

$$V_k = \begin{pmatrix} r_k \\ \alpha_k \\ \varphi_k \end{pmatrix} = \begin{pmatrix} \sqrt{(x_k - x_c)^2 + (y_k - y_c)^2} \\ (\arctan(\frac{y_k - y_c}{x_k - x_c}) \times \frac{180}{\pi} + 360) \% 360 \\ ((\theta_k - \theta_c) + 360) \% 360 \end{pmatrix} \quad (4)$$

where the centroid $P_c = (x_c, y_c, \theta_c)$ is the pole, r_k denotes the polar radius of V_k relative to P_c , α_k denotes the polar angle of V_k relative to P_c , and φ_k denotes the angle of V_k relative to P_c according to their directions.

In the minutiae structure vector LS , radial triangulation vector $RT = \{V_k, R_k, L_{k,k+1}, ST_k\}_{k=1}^N$ is used for local minutiae comparison and the centroid P_c is used for global minutiae comparison.

3. Latent-to-full palmprint comparison

The procedures of latent-to-full palmprint comparison based on radial triangulation are shown in Figure 2. As minutiae-based approach, features of latent prints are manually extracted by forensic experts, while palmprint preprocessing and post-processing are implemented for full palmprint feature extraction by a commercial biometric SDK [11] in an automatic way, which is in accordance with methods in [10] and current forensic practice. Based on the minutiae sets of latent and full palmprints, modified latent-to-full palmprint comparison based on radial triangulation is described as the following sections, mainly improving methods for local minutiae comparison, alignment and comparison score computation.

3.1. Modified radial triangulation comparison

As latent palmprints are partial prints and much smaller than full prints, position and direction information of a minutia cannot be used for latent-to-full palmprint comparison directly. Instead, local minutiae comparison based on radial triangulation is used [10], as described in the following steps while point pattern comparison by relaxation is utilized for the similarity computation in radial triangulation structures with N minutiae.

3.1.1 Selection of local minutiae sets

Given the minutiae set of a palmprint, selection of local N -minutiae sets is detailed as below.

- Step 1: Minutiae are sorted in ascending order of the

distances between minutiae and the origin of the coordinate system.

- Step 2: The minutia nearest to the origin in the sorted minutiae set is selected as the starting point. And $N-1$ minutiae nearest to the starting minutia are selected to form a local N -minutiae set. Then the total N minutiae are deleted from the sorted minutiae set.
- Step 3: The same procedure as the above step is implemented on the reduced minutiae set until the number of remaining minutiae is less than N .

3.1.2 Generation of radial triangulation structures

Local minutiae structure sets $LSS_{Latent} = \{LS_i\}_{i=1}^L$ and $LSS_{Full} = \{LS_j\}_{j=1}^F$ based on radial triangulation are extracted for each latent and full palmprint based on their local N -minutiae sets. $L = \lfloor n_{LM} / N \rfloor$ and $F = \lfloor n_{FM} / N \rfloor$ denote radial triangulation structure numbers in a latent print and a full print, where n_{LM} and n_{FM} denote numbers of minutiae in a latent palmprint and a full palmprint respectively.

3.1.3 Comparison of radial triangulation structures

Comparison between two local radial triangulation structure sets $RTS_{Latent} = \{RT_i\}_{i=1}^L$ from a latent palmprint and $RTS_{Full} = \{RT_j\}_{j=1}^F$ from a full palmprint is executed as below.

- Step 1: For each pair of radial triangulation structures RT_i and RT_j , their distance vector D_{ij} is calculated by

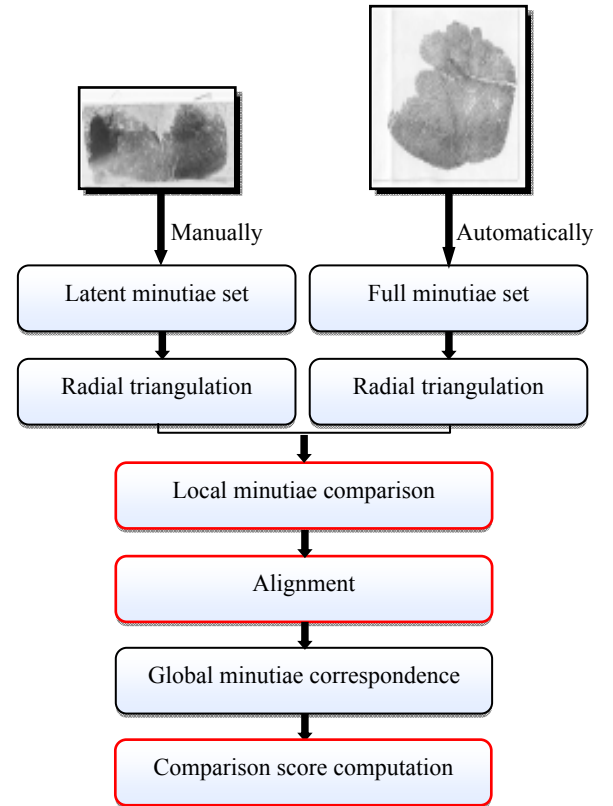


Figure 2: Latent-to-full palmprint comparison.

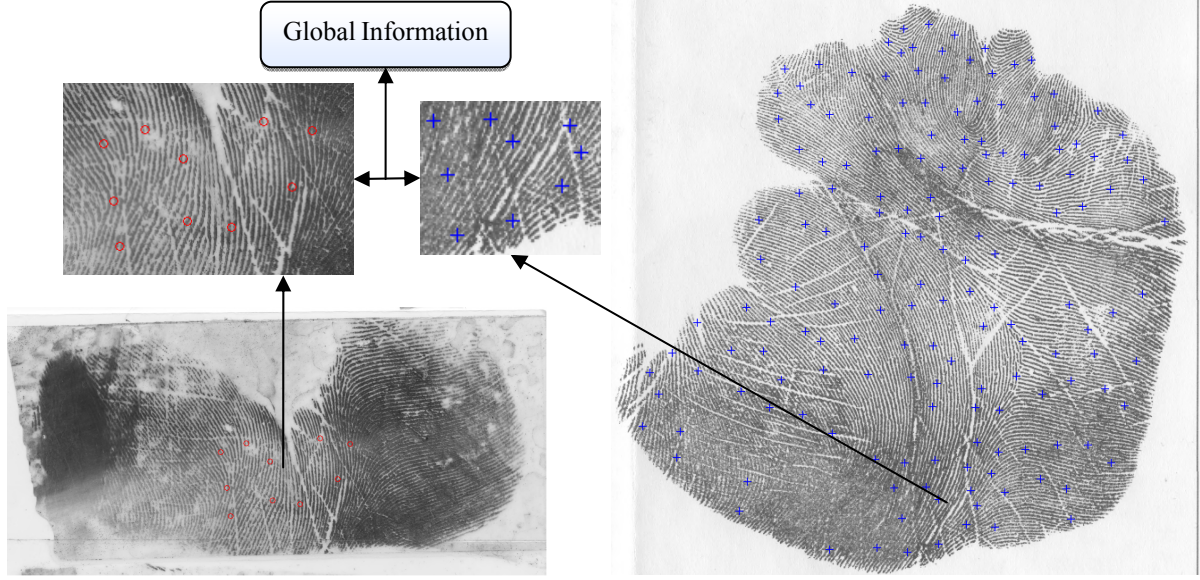


Figure 3: Alignment based on centroid distribution.

(5), while the pair of the structure is considered as a matched pair when the distance vector is within the threshold of the average distance, i.e.,

$$D_0 = \{DV_0, DR_0, DL_0, DST_0\}.$$

$$D_{ij} = \left\{ \frac{1}{N^2} \sum_{l=1}^N \sum_{k=1}^N DV_{i_k, j_l}, \frac{1}{N^2} \sum_{l=1}^N \sum_{k=1}^N DR_{i_k, j_l}, \frac{1}{N^2} \sum_{l=1}^N \sum_{k=1}^N DL_{i_k, j_l}, \frac{1}{N^2} \sum_{l=1}^N \sum_{k=1}^N DST_{i_k, j_l} \right\} \quad (5)$$

where $DV_{i_k, j_l} = \sqrt{(r_{i_k} \cos \alpha_{i_k} - r_{j_l} \cos \alpha_{j_l})^2 + (r_{i_k} \sin \alpha_{i_k} - r_{j_l} \sin \alpha_{j_l})^2}$,

$DR_{i_k, j_l} = |r_{i_k} - r_{j_l}|$, $DL_{i_k, j_l} = |L_{i_k} - L_{j_l}|$, $DST_{i_k, j_l} = |ST_{i_k} - ST_{j_l}|$.

- Step 2: Then the similarity $s_M(LM_{i_k}, FM_{j_l})$ of minutiae LM_{i_k} from the latent print and FM_{j_l} from the full print in the paired structure is calculated using (6), which is deduced according to the similarity of point pattern comparison by relaxation using (2).

$$s_M(LM_{i_k}, FM_{j_l}) = 1/(N-1) \sum_{\substack{p \neq k \\ q \neq l}} \{ \max[\varphi(|\delta_{i_k, j_l}(p, q)|)] \}. \quad (6)$$

where $\delta_{i_k, j_l}(p, q)$ denotes the position difference of LM_{i_p} and FM_{j_q} when LM_{i_k} is mapped into FM_{j_l} , and $\varphi(x) = 1/(1+x^2)$.

- Step 3: The minutia FM_{j_l} is considered as the one paired with the minutia LM_{i_k} only when the similarity $s_M(LM_{i_k}, FM_{j_l})$ is the maximum value in the similarity set of minutia LM_{i_k} , i.e., $s(LM_{i_k}) = \{s_M(LM_{i_k}, FM_{j_l})\}_{l=1}^N$. Finally, all paired minutiae are marked as candidates.

3.2. Global minutiae comparison

Given the similarity of all candidate minutia pairs, global

minutiae correspondence is established in this stage. As the distribution of centroids in radial triangulation structures gives global information, we first do alignment based on centroid distribution and finally obtain the correspondence between minutiae.

3.2.1 Alignment based on centroid distribution

Given a latent and full palmprint with centroid sets $P_{C_{Latent}} = \{P_{C_l}\}_{l=1}^L$ and $P_{C_{Full}} = \{P_{C_j}\}_{j=1}^F$ based on radial triangulation, $L = \lfloor n_{LM} / N \rfloor$ and $F = \lfloor n_{FM} / N \rfloor$ are centroid numbers. As shown in Figure 3, alignment based on centroid distribution is implemented using simplified radial triangulation comparison for centroid sets as following.

- Step 1: Local L -centroid sets for the full palmprint are selected using the same procedure in Section 3.1.2, while only one L -centroid set for a latent palmprint.
- Step 2: Simplified radial triangulation structure sets $LSC_{Full} = \{LSC_l\}_{l=1}^C$ are extracted for the full palmprint based on their L -centroid sets in accordance with the simplified structure form,

$$LSC = [P_{CC}, \{V_k, R_k\}_{k=1}^L] \quad (7)$$

where $P_{CC} = (x_{CC}, y_{CC}, \theta_{CC})$ is the centroid of the polygon formed by a L -centroid set. And the structure number for the full print is $C = \lfloor F/L \rfloor$, while only one simplified radial triangulation structure set is generated for the latent palmprint, i.e., $LSC_{Latent} = [P_{CC}, \{V_k, R_k\}_{k=1}^L]_{Latent}$.

- Step 3: For each simplified radial triangulation structure set in the full palmprint, the similarity $s_C(P_{C_m}, P_{C_n})$ of centroids P_{C_m} from the latent print and P_{C_n} from the full print is calculated by applying the function (6) for minutiae to centroids. Then the support $p(P_{C_m}, P_{C_n})$ of the centroid pair (P_{C_m}, P_{C_n}) giving to the

structure pair (LSC_{Latent}, LSC_l) , $l=1,2,\dots,C$, is calculated using (8), which will be assigned to the minutiae similarity $s_M(LM_{i_k}, FM_{j_l})$ to obtain modified minutiae similarity $s_0(LM_{i_k}, FM_{j_l})$ using (9).

$$p(P_{C_m}, P_{C_n}) = s_C(P_{C_m}, P_{C_n}) / (\sum_{n=1}^L s_C(P_{C_m}, P_{C_n})) \quad (8)$$

$$s_0(LM_{i_k}, FM_{j_l}) = s_M(LM_{i_k}, FM_{j_l}) \times p(P_{C_m}, P_{C_n}) \quad (9)$$

where the minutiae LM_{i_k} and FM_{j_l} are in the radial triangulation structures related to the centroids P_{C_m} and P_{C_n} respectively.

- Step 4: All minutia pairs are sorted in the decreasing order of the modified similarity defined in Step 3 and each of the top-ten minutia pairs is used to align the two sets of minutiae.

3.2.2 Correspondence of minutiae

After the above alignment, minutiae are examined in turn, while a minutiae pair that is the closest with each other according to both location and direction, is deemed as a matched minutiae pair. After all the minutiae pairs have been examined, a set of matched minutiae is obtained.

3.3. Comparison Score Computation

The minutiae-based comparison score S between two palmprints in this paper is set by a linear expression (10) using a quantitative score S_n and a qualitative score S_q .

$$S = \lambda_0 + \lambda_1 \times S_n + \lambda_2 \times S_q \quad (10)$$

where λ_0 , λ_1 , λ_2 are parameters obtained by logistic regression learning [14].

The quantitative score measures the quantity of evidence while the qualitative score measures the consistency in the common region between two palmprints. The quantitative score is computed as

$$S_n = \frac{N_M}{N_M + 20} \quad (11)$$

where N_M denotes the number of matched minutiae and the value 20 is an experience value of the minimum number of matched minutiae for genuine comparison referred to [2]. The qualitative score is computed as

$$S_q = S_D \times \frac{N_M}{N_M + N_L} \times \frac{N_M}{N_M + N_F} \quad (12)$$

where S_D is the average similarity of radial triangulation structures for all the matched minutiae, N_L and N_F denote the number of unmatched minutiae in latent and full palmprints.

4. Experiments

Since public latent and mated full palmprint databases are not available, in our experiments, 22 latent palmprints

from real forensic cases and 8680 full palmprints from criminal investigation field captured by Beijing Institute of Criminal Technology in China, are used to evaluate the performance of the proposed forensic automatic palmprint identification system. 8680 full prints are from 4340 subjects while each subject leaves two palmprints from left hand and right hand respectively, including mated images to the 22 latent prints. Image size of full palmprints is 2304×2304 , while the size of latent palmprints varies and generally is smaller than that of full prints. All the palmprints are recovered (i.e., latent prints) or captured (i.e., full prints) with the resolution of 500 ppi.

Under forensic conditions, minutiae of 22 latent palmprints are manually extracted by forensic experts, while minutiae extraction of 8680 full palmprints is executed by MegaMatcher 4.0 SDK in an automatic way with preprocessing and post-processing as [10]. This protocol simulates typical casework when automated systems are used for query search.

Using the same databases of 22 latent palmprints and 8680 full palmprints, and the same feature sets extracted in the same way, we compare the proposed latent-to-full palmprint comparison algorithm based on modified radial triangulation with the original radial triangulation based method [10] and MinutiaCode-based method [2]. In the experiments, we set $N=9$ for radial triangulation as the performance indicated in [10]. Score computation based on logistic regression learning which operates with maximum likelihood estimators is executed in the performance evaluation for all the three methods. As stated in [18] that it is risky to use maximum likelihood estimates in samples under 100 while samples above 500 should be adequate, we select 2200 score pairs (S_n, S_q) generated by comparison

between 22 latent palmprints and 100 randomly selected full palmprints as the training set. The values of the learned parameters $(\lambda_0, \lambda_1, \lambda_2)$ are (1.4, 1.2, 2.5) for modified radial triangulation based method, (0.8, 1.6, 1.8) for original radial triangulation based method [10] and (2.2, 0.8, 1.5) for MinutiaCode-based method [2] respectively. To evaluate the closed-set identification task, CMC curves for the three comparison methods on the same databases are obtained as shown in Figure 4. Comparison results of identification rate and efficiency are shown in Table 1. The modified radial triangulation based method performs best, i.e., rank-1 identification rate of 69% is achieved, while the average comparison time is 160ms. It is obvious that the proposed method in our work meets the requirements of forensic automatic palmprint identification system best. Though its average comparison time is a little more than that of the original radial triangulation based method [10], it is mainly for a tradeoff to obtain more identification accuracy while implementing improved local comparison by relaxation and finer alignment based on centroid distribution.

Table 1: Comparison results of identification rate and efficiency.

	Identification rate		Average time
	Rank-1	Rank-20	
MinutiaCode[2]	67%	73%	300ms
RT[10]	63%	70%	100ms
Modified RT	69%	78%	160ms

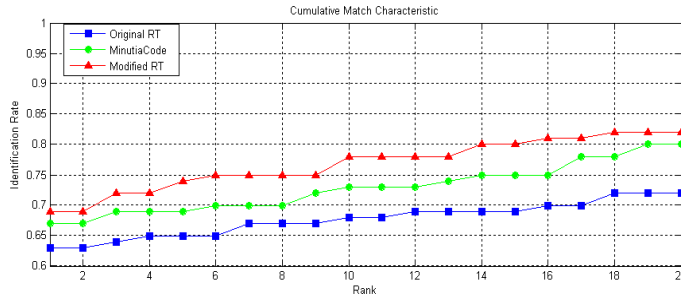


Figure 4: CMC curves of the three methods for comparison.

5. Conclusions

Aimed to improve forensic palmprint recognition technology as a first step towards a rigorous quantification of the evidential value in palmprints, we implemented modified latent-to-full palmprint comparison algorithm. Firstly, we generalized point pattern comparison by relaxation [5] to minutiae-based palmprint recognition and applied it to the similarity computation based on radial triangulation structure comparison. Secondly, we assigned the support of centroid pair to the similarity of minutiae for finer alignment between latent and full palmprints. Finally, logistic regression learning was used for comparison score computation. Forensic databases including 22 latent palmprints from real cases and 8680 full palmprints from criminal investigation field were used for performance evaluation. The modified radial triangulation based method performs best, i.e., rank-1 identification rate of 69% is achieved, while the average comparison time is 160ms. Our future work will focus on quantification of the evidence in palmprints, such as developing a model for the computation of likelihood ratios using the evidence in palmprints.

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